# Predicting the 2004 U.S. Presidential Election

Benjamin Schak

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# 0 Introduction

This year, American voters will choose the next president. The 2000 election was as much a tie as any in history: The electoral college had its second-closest result since the 12<sup>th</sup> Amendment reforms of 1804<sup>1</sup>, the two-party popular vote has only been closer by percentages in 1880<sup>2</sup> and 1960, the popular vote and electoral vote diverged for the third time ever<sup>3</sup>, Gore would have won if just 269 of Bush's Floridian voters had voted for him, and the result was uncertain until mid-December. [Inf,Wik]

The 2004 election figures to be just as close and hard-fought. [Pun] This time, however, we will not have to wait until five weeks after the election to know the winner; rather, this site will (hopefully) predict the winner in advance. Will it be incumbent George Walker Bush (R–Texas) and four more years of paucilateral warfare, tax breaks for Paris Hilton, right-wing judicial nominees? Or Sen. John Forbes Kerry (D–Massachusetts) and an agenda of renewed fiscal

<sup>&</sup>lt;sup>1</sup>In the disputed 1876 election Rutherford Hayes beat Samuel Tilden 185–184.

<sup>&</sup>lt;sup>2</sup>In the seldom discussed 1880 election, James Garfield beat Winfield Hancock 214–155 in the Electoral College, but won the popular vote by only 7018 votes. The possibility that such a close popular vote should occur again should serve as a warning to those who advocate direct national election of the President. Just imagine Florida-style recounts in all 50 states with miscellaneous degrees of competence and corruption!

<sup>&</sup>lt;sup>3</sup>I count 1876, 1888, and 2000 as the only such elections, although it is hardly clear that the electoral vote was decided correctly in 1876 or 2000. Some people would include 1824 in this list. In 1824, there was in fact *no* winner in the Electoral College (Andrew Jackson led John Quincy Adams, William Crawford, and Henry Clay), and it was the House of Representatives that chose Adams. It is often claimed that Jackson won the 1824 popular vote, but this claim is misleading: In 1824, many states (notably New York) chose electors in their legislature, instead of by election; if these states had held elections for president, Adams would have won the national popular vote.

responsibility, membership in the international community, respect for established science, and health care for all? Will Ralph Nader fuck it up again?<sup>4</sup>

How will I<sup>5</sup> predict the election? The key is to look at each state individually. The President and Vice President of the United States are not elected by American voters; rather, each state<sup>6</sup> votes for a slate of electors, who then choose the president. (See the appendix for details.) Because of this, I look at polls in each state and predict the result of each state (essentially) separately. The rest of this document tells what I do to the data to get predictions. It assumes some knowledge of statistical talk, but I don't know how much; I haven't taken a stat course and I can understand most of this document just fine. Expect rambling footnotes, expect semicolons, and expect a dilettante's command of the field of statistics.

### 1 Combining Data

My basic strategy will be to combine data from polls to form predictions for each state. So how does one combine statistical information?

Suppose we have two polls. Poll 1 has 532 responses for candidate K and 511 for B, and Poll 2 has 870 for K and 895 for B. Common sense tells us that we can combine these polls by adding together the responses from each poll for each candidate.<sup>7</sup> Thus, K has 1402 responses, and B has 1406.

Let's look closer at this example. Poll 1 gives K a mean percentage of  $k_1 = 532/(532 + 511)$  with variance approximately<sup>8</sup>  $v_1 = (4n_1)^{-1}$ ; Poll 2 gives K a mean percentage of  $k_2 = 870/n_2$  with variance approximately  $v_2 = (4n_2)^{-1}$ . I

<sup>&</sup>lt;sup>4</sup>I promise, none of the following paragraphs is this partisan. If you're a Republican or if you're a Green election-fucker-upper, hang with me.

<sup>&</sup>lt;sup>5</sup>Who am I? I'm a math grad student at the University of Pennsylvania. My email address is benjamin@schak.com.

<sup>&</sup>lt;sup>6</sup>For purposes of this document, "state" includes the District of Columbia, which has three electoral votes.

<sup>&</sup>lt;sup>7</sup>Common sense if often common nonsense. What if one poll is less valid than the other? For example, one poll might be months old, or might include all adults instead of just voters, might be weighted by partisan affiliation. I deal with the time factor in the next section. Think about the others.

<sup>&</sup>lt;sup>8</sup>More precisely,  $v_1 = 532(511)/n_1^3$ . This approximation is valid when K and B have similar response totals, a reasonable assumption everywhere but Washington, D.C. Since I can predict the winner of D.C.'s electoral votes with one hand closed and my eyes tied behind my back, I'm not worried about this assumption.

write the number of responses in these polls as  $n_1$ ,  $n_2$ . Combined, the polls give K a mean percentage of

$$\frac{n_1k_1 + n_2k_2}{n_1 + n_2} = \frac{k_1/(4v_1) + k_2/(4v_2)}{1/(4v_1) + 1/(4v_2)}$$
$$= \frac{k_1v_2 + k_2v_1}{v_1 + v_2}$$

with variance

$$\frac{1}{4(n_1+n_2)} = \frac{1}{1/v_1 + 1/v_2} \\ = \frac{v_1v_2}{v_1 + v_2}$$

Now we can extract the following principle. Suppose we have two pieces of information: Information 1 says that K has proportion  $k_1$  of the vote with variance  $v_1$ , and Information 2 says that K has proportion  $k_2$  of the vote with variance  $v_2$ . Then we can say that K has

$$\frac{k_1 v_2 + k_2 v_1}{v_1 + v_2}$$

of the vote with variance

$$\frac{v_1v_2}{v_1+v_2}$$

For three pieces of information, we get something like<sup>9</sup>

$$\frac{k_1v_2v_3 + k_2v_3v_1 + k_3v_1v_2}{v_2v_3 + v_3v_1 + v_1v_2} \quad \text{and} \quad \frac{v_1v_2v_3}{v_2v_3 + v_3v_1 + v_1v_2}.$$

And similarly for more data.

# 2 Past Data, Present Information

If we have on hand a poll taken today, it is clear how to interpret it: The mean is the mean, and the variance is  $(4n)^{-1}$ . But what if we have a poll taken 30 days ago? The mean is still the mean, but our confidence in this poll as a predictor of current opinion is less than it was back when the poll was fresh. The way we express this observation is that the variance increases slightly each day.

<sup>&</sup>lt;sup>9</sup>I haven't checked these, but they seem right intuitively, don't they?

The exact amount I use is 0.000001 per day.<sup>10</sup> Thus, a 1043-person poll from 31 days ago has a variance of  $1/(4(1043)) + 0.000031 \approx 0.0002707$ ; the same poll from 183 days ago has a variance of  $1/(4(1043)) + 0.000183 \approx 0.0004227$ . The corresponding standard deviations are 1.6% and 2.1%.

### 3 Kalman Filter and Smoothing

The Kalman Filter provides a simple and efficient way to combine past data from many different dates. We want to get an estimate of support for candidate K today. We have two pieces of information at our disposal: Yesterday's estimate (the algorithm is iterative or recursive) and today's polls. In the language of two section's ago,  $k_1$  is the mean from yesterday,  $v_1$  is 0.000001 plus the variance from yesterday,  $k_2$  is the mean from today's aggregated polls, and  $v_2$  is the variance from today's aggregated polls. We put all this together for today's estimate using the formulas from two sections ago. That's basically it.

The previous paragraph was simplified. Here's the more complex version: Suppose we have a whole vector of statistics to track (say, proportion of responses for K in each state). The notation and approach here are hybrids of the discussions in [Koo] and [And].

Let  $\alpha_t$  denote the underlying vector of actual support for K at time *i*.  $\alpha_t$  changes over time according to

$$\alpha_t = T\alpha_{t-1} + H\epsilon.$$

( $T_t$  is essentially the identity except for some alterations for Maine and Nebraska discussed in the next section, and  $\epsilon_t$  is a vector of independent standard normal random variables.) Now, instead of adding 0.000001 of variance each day, we add the variance *matrix HH*<sup>'</sup>.<sup>11</sup>

On day *t* we have an observation (aggregated new polls)  $y_t$  related to  $\alpha_t$  as

$$y_t = Z\alpha_t + G_t\epsilon.$$

 $<sup>^{10}</sup>$ I tried finding a principled way to establish this constant from the data, but failed in every effort. I tried looking at the unexplained variance in national polls and I tried a maximum likelihood approach, but for various reasons both these approaches failed. This value feels right: It gives results that I find mostly reasonable and says that a national election in year *y* will predict an election between the same candidates four years later with standard deviation 3.69%. That seems about right to me.

<sup>&</sup>lt;sup>11</sup>I represent the transpose of a matrix with the prime symbol '.

*Z* is the identity and  $G_tG'_t$  is analogous to the sample variance of  $y_t$ . On a typical day, we don't get polls from every state. In such a situation we just delete the rows corresponding to missing data from  $y_t$ , *Z*, and *G*.

Now suppose the mean and variance obtained at time t - 1 were  $k_{t-1}$  and  $V_{t-1}$ . Let

$$K = (TV_{t-1}T' + HH')Z'(Z(TV_{t-1}T' + HH')Z' + GG')^{-1}.$$

Then

$$k_t = Tk_t + K(y_t - ZTk_t)$$

and

$$V_t = (I - KZ)(TV_{t-1}T' + HH').$$

I haven't actually worked through the proofs of these formulas, but I have checked that they reduce correctly to the univariable case.

I initialize each state based on its 2000 election returns (a poll with thousands to millions of responses depending on the state that still failed to get statistically significant results in a couple places) with the variance adjusted for how long ago that was.

Before moving on to smoothing, one more disorganized comment about the time variance matrix HH'. HH' may change through time. I have HH' constant except that I use 2HH' on convention days (except the final day of each convention), 4HH' on the final day of each convention and the vice presidential debate day, 7HH' on the presidential debate days, and HH', 2HH', ..., 7HH' on the last week of the campaign. This reflects the greater swing potential at these times.

I don't want to dwell long on smoothing. The idea is to integrate date from more recent polls to improve estimates for old dates. This is useful for tracking past movement,<sup>12</sup> but is irrelevant for making future predictions.

I haven't worked through the formulas much myself but parts of the formulas seem intuitive to me, and they seem to work. I write  $k^*$  and  $V^*$  for the smoothed estimates. Let  $A = V_t T' (TV_t T + HH')^{-1}$ . Then  $k_t^* = k_t + A(k_{t+1}^* - Tk_t)$  and  $V_t^* = V_t + A(V_{t+1}^* - TV_t T' - HH')A'$ . That's it.

<sup>&</sup>lt;sup>12</sup>I can generate some awesome graphs of various statistics through time but can't export them yet.

### 4 The Nader Effect

I believe Nader is only likely to be on the ballot in a handful of states<sup>13</sup>, but pollsters seem to include or exclude him in polls without regard to his ballot access. To handle this I need to estimate the Nader effect and then add or subtract the Nader effect from polling data as appropriate.

First I perform Kalman filtering and smoothing on all national polls<sup>14</sup> including Nader and all national polls excluding him separately. From the result of this, I calculate the (obviously rather high) correlation  $\gamma$  between the daily change in Naderless opinion and the daily change in Naderly<sup>15</sup> opinion. I set *H* to be

$$v \begin{pmatrix} 1 & \gamma \\ \gamma & 1 \end{pmatrix}$$

and run the multivariate Kalman filter and smoothing with Naderless polls in the first variable and Naderly polls in the second. This gives me, for each time t, Kerry's Naderless proportion  $k_t^+$  and Naderly proportion  $k_t^-$  of the two-party vote, with respective variances  $V_t^+$  and  $V_t^-$ . Let  $d_t = k_t^+ - k_t^-$ . The variance of  $d_t$  is  $V_t^+ + V_t^-$ . For the mean of the Nader effect, I take the average (over t) of the  $d_t$ . For the variance of the Nader effect, I take the variance (over t) of the  $d_t$  and then add the average (over t) of the statistic  $V_t^+ + V_t^-$ .

Then, when I see a Naderly poll in a Naderless state, I add the Nader effect to that poll's Kerry proportion and I add the Nader effect variance to that poll's variance. And *vice versā* for a Naderless poll in a Naderly state.

### 5 Maine and Nebraska

Maine and Nebraska have a unique<sup>16</sup> way of divvying up their electoral votes. Instead of the statewide winner getting all the state's electoral votes, the statewide winner gets 2, and the winner of each Congressional district gets 1. Maine has two districts and Nebraska has three.

<sup>&</sup>lt;sup>13</sup>Between his status as Reform Party nominee and easy ballot access in many states, I project that Nader will be on the ballot in Colorado, Florida, Kansas, Louisiana, Michigan, Minnesota, Mississispipi, Montana, New Jersey, South Carolina, Tennessee, Washington, and Wisconsin.

<sup>&</sup>lt;sup>14</sup>Polls since March 1<sup>st</sup> only.

<sup>&</sup>lt;sup>15</sup>The opposite of Naderless.

<sup>&</sup>lt;sup>16</sup>"Unique" is absolutely the wrong word. Is there a word for something with only one example? "Bi-unique"? "Semi-unique"? "Almost unique"?

Because of this I adjust *HH*' and *Z*. I'm sure this all seemed inspired at one point, but now this just reminds me of the old line about how one doesn't really want to know how sausages are made. Anyhow, let p, q, r be the proportions of Nebraska's two-party from each Congressional district, let  $k_1$ ,  $k_2$ ,  $k_3$  be the proportions of the two-party vote that Gore got in each district, and let v = 0.000001. Then the 4 × 4 Nebraska portion of *HH*' is the symmetric matrix

$$v \begin{pmatrix} * & & \\ * & 1 & & \\ * & .3 - .5|k_1 - k_2| & 1 & \\ * & .3 - .5|k_1 - k_3| & .3 - .5|k_2 - k_3| & 1 \end{pmatrix}$$

where the first column of asterisks is

$$\begin{pmatrix} p^2 + q^2 + r^2 + \\ + 2pq(.3 - .5|k_1 - k_2|) + 2pr(.3 - .5|k_1 - k_3|) + 2qr(.3 - .5|k_2 - k_3|) \\ p + q(.3 - .5|k_1 - k_2|) + r(.3 - .5|k_1 - k_3|) \\ p(.3 - .5|k_1 - k_2|) + q + r(.3 - .5|k_2 - k_3|) \\ p(.3 - .5|k_1 - k_3|) + q(.3 - .5|k_2 - k_3|) + r \end{pmatrix}.$$

And similarly for Maine. This comes from  $k_{NE} = pk_{NE1} + qk_{NE2} + rk_{NE3}$ . We have, for example,

$$Var(k_{NE}) = Var(pk_{NE1} + qk_{NE2} + rk_{NE3})$$
  
=  $p^2 Var(k_{NE1}) + q^2 Var(k_{NE2}) + r^2 Var(k_{NE3}) +$   
+  $2pq Cov(k_{NE1}, k_{NE2}) + 2pr Cov(k_{NE1}, k_{NE2}) +$   
+  $2qr Cov(k_{NE2}, k_{NE3})$ 

and

$$\operatorname{Cov}(k_{NE}, k_{NE1}) = p \operatorname{Var}(k_{NE1}) + q \operatorname{Cov}(k_{NE1}, k_{NE2}) + r \operatorname{Cov}(k_{NE1}, k_{NE3}).$$

The covariances between the Congressional districts are done similarly to the covariances between states.

The Nebraska row of *Z* has a 0 in the Nebraska column, *p* in the Nebraska<sub>1</sub> column, *q* in the Nebraska<sub>2</sub> column, and *r* in the Nebraska<sub>3</sub> column. And similarly for Maine.

Moving right along...

## 6 Correlation Between States

One problem I face is a paucity of data in some states.<sup>17</sup> Indeed, I currently lack any post-2000 data from several states<sup>18</sup>

The fundamental observation for working around this problem is that similar states will vary similarly; if Kerry gains ground in Alabama (polled four times in May) then he probably gained ground in Mississippi (polled once in April) as well. This relation shows up in the off-diagonal elements of the time variance matrix HH'. To be precise, suppose states *i* and *j* have variances-ofdaily-change  $v_i$  and  $v_j$  (these will be 0.000001 for every state but Maine and Nebraska, which are given above), and suppose their daily change has correlation  $\gamma$ . Then, by the definition of correlation, the covariance we put in the (i, j)<sup>th</sup> and (j, i)<sup>th</sup> elements of HH' should  $\gamma_{\sqrt{v_i v_j}}$ .

So how do I estimate the correlation between the daily changes of two states? I tried deriving something for the data, but there just wasn't enough data to get any decent information. Trust me, it was a mess. I propose that geographically close states and that politically similar states are correlated. Suppose two states have centers of population 500 miles apart and that Gore's share of the two-party vote in the two states was 3% apart. Then I give the states a correlation of 0.2 - 500/20000 for geographic closeness and 0.1 - 0.03/2 for political similarity. That's a total of  $\gamma = 0.26$ . The highest possible  $\gamma$  is a modest 0.3 and the lowest possible is 0.

### 7 Formulating Results

After running the Kalman Filter on all 56 pseudo-states<sup>19</sup> with my correlated H and Maine/Nebraska-tweaked Z, I get (among plenty of other interesting data) a vector k showing the Election-Day mean for each pseudo-state, and a variance matrix V for that vector showing the variance for each pseudo-state

<sup>&</sup>lt;sup>17</sup>Jul. 2<sup>nd</sup> note to national pollsters: I really, really want more New Mexico and Virginia data right now. I didn't need the third Kansas poll in five months that you just did or a dozenth California poll. New Mexico and Virginia, right now.

<sup>&</sup>lt;sup>18</sup>Alaska, Delaware, District of Columbia, Hawaii, Idaho, Nebraska, North Dakota, and Wyoming. Luckily, 2000 was enough of a wipeout in these states that I can predict each one with very good confidence.

<sup>&</sup>lt;sup>19</sup>The 50 states, D.C., and the Congressional districts of Maine and Nebraska.

and the covariance between each pair of pseudo-states. More useful than *V* is the positive definite matrix *A* (I call it the standard deviation matrix) such that V = AA'; AA' is just the Cholesky decomposition of *V*.

To simulate Election Day, I sample 56 numbers independently with mean 0 and standard deviation 1, and put these numbers into a vector r. Then k + Ar represents an Election Day result and I can test whether it represents a Kerry or Bush victory. I perform 9604 simulations<sup>20</sup> and report the probability of Kerry or Bush winning.<sup>21</sup> Using today's variance matrix V, I can also get the probability that Kerry or Bush would win today.

It's straightforward to get the exact expected value of Kerry or Bush's electoral votes from all the state's means and variances, and the estimated popular vote from all the state's means.

I classify and list each state as follows: Close Tossup (50–55% chance of victory for the leader), Tossup (55–60%), Probable (60–70%), Very Probable (70–80%), Safe (80–90%), Very Safe (90–99%), and Extremely Safe (99–100%). I hope that almost every state is in the Very or Extremely Safe category by Election Day.

#### 8 Cartograms

Cartograms rule. A cartogram is, generally, a map showing statistical information. This website has cartograms which show the states of the United States with area proportional to their votes in the 2004 and 2008 Electoral Colleges. To make my cartograms, I make each state out of square blocks (each square is one elector) and just eyeball how they fit together.<sup>22</sup> "Edwards for Prez" has an interactive cartogram<sup>23</sup> which is neat and has quite a different look from mine. When I was little, I loved to look at the *State of the World Atlas*<sup>24</sup> from the Franklin Library, which is filled with cartograms.

<sup>&</sup>lt;sup>20</sup>Why 9604? I had a reason, but can't remember it for the life of me. The number 95% had something to do with it.

<sup>&</sup>lt;sup>21</sup>This Monte Carlo approach leaves something to be desired. I'm thinking about other ways of getting at this.

<sup>&</sup>lt;sup>22</sup>I think I've done a good job. By comparison, the Wall Street Journal's cartogram at http://online.wsj.com/public/resources/documents/info-battleground04-frameset. html (click the "Electoral College" tab) is utter crap.

<sup>&</sup>lt;sup>23</sup>http://www.edwardsforprez.com/map.html

<sup>&</sup>lt;sup>24</sup>http://www.amazon.com/exec/obidos/ASIN/0142003182/schak-20/

### **9** Software Notes

I use an Excel knockoff called ThinkFree Calc for some secondary record-keeping, but have moved to using Maple on my office computer for all calculations. I use TextEdit to store my Maple scripts. It takes about ten minutes to run everything through Maple.

I wrote this document with iT<sub>E</sub>XMac, a Macintosh implementation of LAT<sub>E</sub>X that always outputs PDFs. How cool is that? The fonts are Palatino, with Computer Modern Sans-Serif (in the headers and footers) and Courier for the URLs.

### A The Presidential Election Process

The process of electing the President is described by [Con] as below. For more information on the Electoral College, see [Kim].

#### Article II

**Section 1.** The executive Power shall be vested in a President of the United States of America. He shall hold his Office during the Term of four Years, and, together with the Vice President<sup>25</sup>, chosen for the same term, be elected, as follows:

Each State shall appoint, in such Manner as the Legislature thereof may direct<sup>26</sup>, a Number of Electors, equal to the whole Number of Senators and Representatives to which the State may be entitled in the Congress; but no Senator or Representative, or Person holding an Office of Trust or Profit under the United States, shall be appointed an Elector....

<sup>&</sup>lt;sup>25</sup>The Vice President does two things: He is President of the Senate and breaks ties there; and he becomes President or acts as President when the President dies (becomes), resigns (becomes), is expelled from office (becomes) or inability (acts as). For the last possibility, see [Sur] or the 25<sup>th</sup> Amendment.

<sup>&</sup>lt;sup>26</sup>Each state except Maine and Nebraska assigns all its electors to the winner of a statewide popular election. Maine and Nebraska both elect two such at-large electors, and one from each Congressional district. Colorado has a proposal on its ballot this year to choose electors in rough proportion to the presidential vote there; that proposal *would* take effect for this election. In practice, since Colorado has 9 electoral votes, Bush would get 4, Kerry would get 4, and the winner (probably Bush) would get one more; because this would obviously diminish the importance of Colorado's votes, I believe this initiative will fail.

The Congress may determine the Time of chusing the Electors<sup>27</sup>, and the Day on which they shall give their Votes<sup>28</sup>; which Day shall be the same throughout the United States.

#### Amendment XII

The Electors shall meet in their respective states and vote by ballot for President and Vice President, one of whom, at least, shall not be an inhabitant of the same state with themselves; they shall name in their ballots the person voted for as President, and in distinct ballots the person voted for as Vice-President, and they shall make distinct lists of all persons voted for as President, and of all persons voted for as Vice-President, and of the number of votes for each, which lists they shall sign and certify, and transmit sealed to the seat of the government of the United States, directed to the President of the Senate<sup>29</sup>;—The President of the Senate shall, in their presence of the Senate and House of Representatives, open all the certificates and the votes shall then be counted,<sup>30</sup>—The person having the greatest number of votes for President, shall be the President, if such number be a majority of the whole number of Electors appointed; and if no such number be a majority of the whole number of Electors appointed; and if no person have such majority, then from the persons having the highest numbers not exceeding three on the list of those voted for as President, the House of Representatives shall choose immediately, by ballot, the President. But in choosing the President, the votes shall be taken by states, the representation from each state having one vote; a quorum for this purpose shall consist of a member or members from two-thirds of the states, and a majority of all the states shall be necessary to a choice.... The person having the greatest number of votes as Vice-President, shall be the Vice-President, if such number be a majority of the whole number of Electors appointed, and if no person have a majority, then

 <sup>&</sup>lt;sup>27</sup>The first Tuesday after the first Monday of November in years divisible by four. [Kim]
<sup>28</sup>The first Monday after the second Wednesday in December following the popular election.
[Kim]

<sup>&</sup>lt;sup>29</sup>I.e., the Vice President of the United States.

<sup>&</sup>lt;sup>30</sup>This happens on Jan. 6<sup>th</sup> after the election.

from the two highest numbers on the list, the Senate shall choose the Vice-President; a quorum for the purpose shall consist of twothirds of the whole number of Senators, and a majority of the whole number shall be necessary to a choice. But no person constitutionally ineligible to the office of President shall be eligible to that of Vice-President of the United States.

#### Amendment XX

**Section 3.** If, at the time fixed for the beginning of the term of the President, the President elect shall have died, the Vice President elect shall become President. If a President shall not have been chosen before the time fixed for the beginning of his term, or if the President elect shall have failed to qualify, then the Vice President elect shall act as President until a President shall have qualified; and the Congress may by law provide for the case wherein neither a President elect nor a Vice President elect shall have qualified, declaring who shall then act as President, or the manner in which one who is to act shall be selected, and such person shall act accordingly until a President or Vice President shall have qualified.

**Section 4.** The Congress may by law provide for the case of the death of any of the persons from whom the House of Representatives may choose a President whenever the right of choice shall have devolved upon them, and for the case of the death of any of the persons from whom the Senate may choose a Vice President whenever the right of choice shall have devolved upon them.

#### Amendment XXII

**Section 1.** No person shall be elected to the office of the President more than twice, and no person who has held the office of President, or acted as President for more than two years of a term to which some other person was elected President shall be elected to the office of President more than once....

#### Amendment XXIII

**Section 1.** The District constituting the seat of Government of the United States<sup>31</sup> shall appoint in such manner as the Congress may direct:

A number of electors of President and Vice President equal to the whole number of Senators and Representatives in Congress to which the District would be entitled if it were a State, but in no event more than the least populous state; they shall be in addition to those appointed by the States, but they shall be considered, for the purposes of the election of President and Vice President, to be electors appointed by a state; and they shall meet in the District and perform such duties as provided by the twelfth article of amendment.

#### Amendment XXIV

**Section 1.** The right of citizens of the United States to vote in any primary or other election for President or Vice President, [or] for President and Vice President, ... shall not be denied or abridged by the United States or any State by reason of failure to pay any poll tax or other tax.

#### Amendment XXVI

**Section 1.** The right of citizens of the United States, who are eighteen years of age or older, to vote shall not be denied or abridged by the United States or by any State on account of age.

#### References

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<sup>&</sup>lt;sup>31</sup>I.e., Columbia.

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